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0301207C1010 **Seat No :________**

B.Sc. Semester - 2 (*CBCS*) Examination

March/April - 2023 (OLD COURSE)

MATHEMATICS (CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Q.1(A) Answer the following questions. (Any One) **(07)**

1. Find the equation of the tangent plane at any point (α, β, γ) on the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$.
2. Derive equation of a cylinder of which generator parallel to the line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ and passing through a guiding curve $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0;\ z = 0$.

Q.1(B) Answer the following question. (Any One) **(04)**

1. Get the equation of sphere through (2, -1, -1) and (0, -2, -4) and having centre on $x + y + z = 0,\ 5y + 2z = 0$.
2. Find equation of right circular cylinder of which guiding curve is a circle $x^2 + y^2 + z^2 = 4, \quad x + y + z = 3$.

Q.1(C) Answer the following question. (Any One) **(03)**

1. Find the two tangent planes to the sphere $x^2 + y^2 + z^2 - 6x - 4y - 2z + 5 = 0$ which are parallel to the plane $8x + y + 4z + k = 0$.
2. Find the equation of the sphere through the circle $x^2 + y^2 + z^2 = 9, 2x + 3y + 4z = 5$ and the point (1, 2, 3).

Q.2(A) Answer the following questions. (Any One) **(07)**

1. If the function $z = f(x, y)$ possesses the first order partial derivatives in the domain D and $x = g(u, v), y = h(u, v)$ also possesses its first order partial derivatives then show that $\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u}$ and $\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v}$.
2. If $u = \emptyset(x, y)$, where $x = e^a cost, y = e^a sint$ then show that:

   (i) $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = e^{-2a}\left[\left(\frac{\partial u}{\partial a}\right)^2 + \left(\frac{\partial u}{\partial t}\right)^2\right]$ **(ii)** $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = e^{-2a}\left[\frac{\partial^2 u}{\partial a^2} + \frac{\partial^2 u}{\partial t^2}\right]$

Q.2(B) Answer the following question. (Any One) **(04)**

1. If $u = sin(\sqrt{x} + \sqrt{y})$ then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}(\sqrt{x} + \sqrt{y})cos(\sqrt{x} + \sqrt{y})$.
2. Verify Euler's theorem for homogeneous function $u = x^3\emptyset\left(\frac{y}{x}\right)$.

Q.2(C) Answer the following question. (Any One) **(03)**

1. If $(x, y) = \frac{xy}{\sqrt{x^2+y^2}}$; when $(x, y) \neq (0,0)$

   $= 0$; when $(x, y) = (0,0)$

   then discuss the continuity of f at (0,0).
2. If $w = \frac{y}{z} + \frac{x}{y} + \frac{z}{x}$ then prove that $x\frac{\partial w}{\partial x} + y\frac{\partial w}{\partial y} + z\frac{\partial w}{\partial z} = 0$.

Q.3(A) Answer the following questions. (Any One) **(07)**

1. State and prove Taylor's theorem for the function $z = f(x, y)$.
2. Find maxima and minima for the function $f(x, y) = x^2 - 2xy + \frac{y^3}{3} - 3y$.

Q.3(B) Answer the following question. (Any One) **(04)**

1. Find the volume of the greatest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.
2. Expand $e^x siny$ in power of x and y.

Q.3(C) Answer the following question. (Any One) **(03)**

1. If da, db and dc are errors in measurements in a, b and c respectively as length of sides of ΔABC then prove that $\frac{da}{cosA}+\frac{db}{cosB}+\frac{dc}{cosC}=0$.
2. If $u=\frac{x+y}{1-xy}$, $v=tan^{-1}x+tan^{-1}y$ then find $\frac{\partial(u,v)}{\partial(x,y)}$.

Q.4(A) Answer the following questions. (Any One) **(07)**

1. Define: Inverse of a matrix and non-singular matrix. For non-singular matrices A and B, Prove that:
   (i) $(AB)^{-1}=B^{-1}A^{-1}$ **(ii)** A^T is non-singular and $(A^T)^{-1}=(A^{-1})^T$.
2. If $A=\left[a_{ij}\right]_{n\times n}, B=\left[b_{ij}\right]_{n\times n}$ then prove that
   (i) $A(adjA)=(adjA)A=|A|\cdot I_n$ **(ii)** $adj(AB)=adjB\cdot adjA$

Q.4(B) Answer the following question. (Any One) **(04)**

1. Find the rank of the matrix $A=\begin{bmatrix}3 & 1 & 4 & 0\\ 0 & 2 & 2 & 0\\ 1 & -1 & 0 & 0\end{bmatrix}$ by echelon matrix.
2. Define: Hermitian and skew-Hermitian matrix with illustrations.
   (i) If A and B are Hermitian then prove that A+B is Hermitian.
   (ii) If A and B are skew Hermitian then prove that A+B is skew Hermitian.

Q.4(C) Answer the following question. (Any One) **(03)**

1. Define: Involutory matrix. Prove that $A=\begin{bmatrix}0 & 4 & 3\\ 1 & -3 & -3\\ -1 & 4 & 4\end{bmatrix}$ is involutory matrix.
2. Define: Periodic matrix. Find the period of matrix $A=\begin{bmatrix}-1 & -1 & -1\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$.

Q.5(A) Answer the following questions. (Any One) **(07)**

1. Prove that every square matrix satisfies its characteristic equation.
2. Find eigen values and eigen vectors of matrix $A=\begin{bmatrix}2 & 2 & 1\\ 1 & 3 & 1\\ 1 & 2 & 2\end{bmatrix}$.

Q.5(B) Answer the following question. (Any One) **(04)**

1. Solve : $3x+4y+z+2w=3$
   $6x+8y+2z+5w=7$
   $9x+12y+3z+10w=13$
2. Using Cayley-Hamilton theorem, find inverse of $A=\begin{bmatrix}-1 & 2 & -3\\ 2 & 1 & 0\\ 4 & -2 & 5\end{bmatrix}$.

Q.5(C) Answer the following question. (Any One) **(03)**

1. Define: Characteristic equation of a matrix. Find characteristic equation of $A=\begin{bmatrix}0 & c & -b\\ -c & 0 & a\\ b & -a & 0\end{bmatrix}$.
2. Prove that 0 is characteristic root of a matrix A if and only if A is singular.

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